Reg.No. _____

Karunya UNIVERSITY

(Karunya Institute of Technology & Sciences)
(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

End Semester Examination – Nov/Dec – 2016

Semester : 2016-17 ODD
Code : 14MA2001 **Duration : 3hrs**
Sub. Name : Vector Calculus and Complex Analysis **Max. marks : 100**

Q. No.	Questions				Course outcome	Marks
	PART-A (40X1=40 MULTIPLE CHOICE QUESTIONS)					
1.	The speed of a particle whose velocity $\vec{V}=\vec{i}-\vec{j}+3\vec{k}$ is				CO1	
	a. $\sqrt{11}$	b. $-\sqrt{71}$	c. $\sqrt{81}$	d. $-\sqrt{17}$		(1)
2.	The acceleration of a particle, whose path given by $\vec{r}(t)=2e^{-t}\vec{i}+3e^{-t}\vec{j}$ at $t=0$ is				CO1	
	a. $3\vec{i}+2\vec{j}$	b. $3\vec{i}-2\vec{j}$	c. $-3\vec{i}-2\vec{j}$	d. $2\vec{i}+3\vec{j}$		(1)
3.	Derivative of a constant vector is _____.				CO1	
	a. constant	b. vector	c. zero	d. scalar		(1)
4.	A vector $\vec{F}$ is said to be solenoidal if				CO1	
	a. $curl\vec{F}=0$	b. $div\vec{F}=0$	c. $divcurl\vec{F}=0$	d. $gradf=0$		(1)
5.	$\nabla r^n=$_____.				CO1	
	a. nr^{n-2}	b. $nr^{n-2}\vec{r}$	c. nr^2	d. $(n-2)r^{n-2}$		(1)
6.	If $\vec{F}$ is conservative, then _____.				CO1	
	a. $div\vec{F}=0$	b. $curl\vec{F}=0$	c. $divcurl\vec{F}=0$	d. $divdiv\vec{F}=0$		(1)
7.	For any vector $\vec{F}$, $divcurl\vec{F}$ is _____.				CO1	
	a. $div\vec{F}$	b. $curl\vec{F}$	c. 0	d. 1		(1)
8.	If $\vec{F}=6x\vec{i}-2y\vec{j}+4z\vec{k}$, $div\vec{F}$ is _____.				CO1	
	a. 8	b. 1	c. 5	d. 2		(1)
9.	For any scalar function φ, $curl\ grad\varphi$ is _____.				CO1	
	a. vector	b. 1	c. scalar	d. 0		(1)
10.	$\int_C Mdx+Ndy=$_____.				CO1	
	a. $\iint_R\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dxdy$	b. $\int_R\left(\frac{\partial N}{\partial x}+\frac{\partial M}{\partial y}\right)dxdy$	c. $\iint_R\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dxdy$	d. $\iiint_V\left(\frac{\partial N}{\partial x}+\frac{\partial M}{\partial y}\right)dxdy$		(1)
11.	$\iint_S \vec{F}.\hat{n}\,ds=$_____.				CO1	
	a. $\int_C \vec{F}.d\vec{r}$	b. $\iint_S curl\vec{F}.\hat{n}ds$	c. $\iiint_V curl\vec{F}.dv$	d. $\iiint_V div\vec{F}.dv$		(1)
12.	Divergence theorem relates a surface integral in space with _____.				CO1	
	a. line integral	b. surface integral	c. volume integral	d. none		(1)
13.	For a plane curve torsion $\tau=$ _____.				CO1	
	a. constant	b. vector	c. scalar	d. zero		(1)

14.	If $\vec{A}=curl\vec{F}$, then $\iint_C \vec{A}.\hat{n}\,ds$ is _____.				CO1	
	a. n^2	b. $div\vec{F}$	c. $curl\vec{F}$	d. 0		(1)
15.	$\int_C \vec{F}.d\vec{r}$ = _____.				CO1	
	a. $\iint_S curl\vec{F}.ds$	b. $\iint_S curl\vec{F}.\hat{n}ds$	c. $\iint_R \vec{F}.\hat{n}d\vec{r}$	d. $\int_C \vec{F}.\hat{n}ds$		(1)
16.	If $\vec{r}$ is the position vector of a point $P(x,y,z)$, then $div\vec{r}$ =_____.				CO1	
	a.0	b. 3	c. r^2	d. r		(1)
17.	CR equations in Cartesian form are				CO2	
	a. $u_x=v_y, u_y=-v_x$	b. $u_y=v_y, u_x=-v_x$	c. $u_y=-v_y, u_x=-v_x$	d. $u_x=-v_y, u_y=v_x$		(1)
18.	CR equations in Polar form are				CO2	
	a. $u_r=v_\theta, u_\theta=v_r$	b. $ru_r=v_\theta, rv_r=u_\theta$	c. $ru_r=v_\theta, rv_r=-u_\theta$	d. $ru_r=-v_\theta, rv_r=-u_\theta$		(1)
19.	The function $f(z)=\log z$ is analytic everywhere except at				CO2	
	a. $z=r$	b. $z=0$	c. $z=1$	d. $z=\infty$		(1)
20.	If u(x, y) is harmonic , then				CO2	
	a. $u_{xx}-u_{yy}=0$	b. $u_{xx}+u_{yy}=0$	c. $u_x+u_y=0$	d. $v_x-v_y=0$		(1)
21.	The mapping $w=cz$, c is a real constant is called _____.				CO3	
	a. inversion	b. translation	c. inversion	d. magnification		(1)
22.	The mapping $w=\frac{1}{z}$ is called _____.				CO3	
	a. magnification	b. rotation	c. translation	d. inversion		(1)
23.	The mapping $w=c+z$, c is a complex constant is called _____.				CO3	
	a. translation	b. magnification	c. rotation	d. inversion		(1)
24.	Under the mapping $w=c+z$ straight lines are mapped onto _____.				CO3	
	a. ellipses	b. parabolas	c. straight lines	d. circles		(1)
25.	Under the mapping $w=\frac{1}{z}$ straight lines are mapped onto _____.				CO3	
	a. straight lines	b. circles	c. ellipses	d. parabolas		(1)
26.	The fixed points of $w=\frac{1}{z}$ are _____.				CO3	
	a. $z=\pm i$	b. $z=0,1$	c. $z=\pm 1$	d. $z=\pm 2$		(1)
27.	If $w=3z$, then u = _____.				CO3	
	a. x + y	b. 2y	c. 3x	d. x/y		(1)
28.	If $w=z+2+i$, then v = _____.				CO3	
	a. x+1	b. y+1	c. x + y	d. none		(1)
29.	The critical point of the mapping $w=z^2$ is _____.				CO3	
	a. z = 0	b. z = 1	c. z = -1	d. $z=\infty$		(1)
30.	A mapping which preserves the angle both in magnitude and direction is called_____.				CO3	
	a. bilinear	b. conformal	c. isogonal	d. none		(1)

31.	A mapping which preserves the angle only in magnitude is called_____.				CO3	
	a. isogonal	b. bilinear	c. conformal	d. none		(1)
32.	If $f(z)$ is analytic inside and on a simple closed curve C, then $\oint_C f(z)dz$ = _____.				CO3	
	a. 1	b. $2\pi i$	c. 0	d. ∞		(1)
33.	$\oint_C \frac{z+1}{z-2}dz$ = _____, where C is $\lvert z\rvert = 1/2$.				CO3	
	a. 0	b. 1	c. 2π	d. 4π		(1)
34.	The point $z=-1$ lies_____ the circle $\lvert z+1\rvert=2$.				CO3	
	a. inside	b. outside	c. on	d. none		(1)
35.	The centre of the circle $\lvert z-i\rvert=1$ is _____.				CO3	
	a. (3 , 0)	b. (0 , 1)	c. (0 , 3)	d. (2 , 2)		(1)
36.	The poles of $f(z)=\frac{z}{(z-2)(z+1)}$ are				CO3	
	a. $z=2,3$	b. $z=-2,-3$	c. $z=-2,3$	d. $z=2,-1$		(1)
37.	If $f(z)$ is analytic inside and on C and $z=a$ lies inside C, then $\oint_C \frac{f(z)}{z-a}dz$ =_____.				CO3	
	a. $2\pi i\, f(a)$	b. $2\pi i\, f'(a)$	c. $2\pi i\, f''(a)$	d. none		(1)
38.	$\oint_C \frac{f(z)}{(z-a)^2}dz$, where $z=a$ is a point which lies inside C is _____.				CO3	
	a. $2\pi i\, f(a)$	b. $2\pi i\, f'(a)$	c. $2\pi i\, f''(a)$	d. none		(1)
39.	If $w=z^2$, then v = _____.				CO3	
	a. x^2-y^2	b. $2xy$	c. x^2+y^2	d. $-2xy$		(1)
40.	$\oint_C \frac{e^z}{z-2}dz$, where C is $\lvert z\rvert=3$ is _____.				CO3	
	a. $2\pi i e$	b. $2\pi i e^2$	c. $\frac{2\pi i}{e^2}$	d. $-\frac{2\pi i}{e^2}$		(1)

	PART B (8 X 5 = 40 MARKS) (ANSWER ANY EIGHT)		
41.	Find the component of velocity, speed, acceleration of a particle whose path is given by $\vec{r}(t)=4t\vec{i}-4t\vec{j}+t\vec{k}$ in the direction of $\vec{i}+\vec{j}+\vec{k}$ at $t=1$.	CO1	(5)
42.	Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at $(2,-1,2)$.	CO1	(5)
43.	Find the work done in moving the particle in the force field $\vec{f}=(5xy-6x^2)\vec{i}-(2y-4x)\vec{j}$ along the curve $y=x^3$ from $x=1$ to $x=2$.	CO1	(5)
44.	Show that the vector field $\vec{f}=(y+z)\vec{i}+(z+x)\vec{j}+(x+y)\vec{k}$ is irrotational and find its scalar potential.	CO1	(5)
45.	Check whether the function $f(z)=e^z$ is analytic or not.	CO2	(5)
46.	Check whether $u=x^2-y^2-2xy-2x+3y$ is harmonic or not.	CO2	(5)
47.	Find the image of $x=0, y=0, x=2, y=1$ under the mapping $w=z+(2+3i)$.	CO3	(5)
48.	Find the Bilinear linear transformation which maps the points $z=\infty, i, 0$ onto $w=-1,-i,1$.	CO3	(5)

49.	Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $\|z\| = 3$.	CO3	(5)
50.	Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in Laurent's series in the region $\|z\| > 2$.	CO3	(5)
	PART C (2 X 10 = 20 MARKS) (ANSWER ANY TWO)		
51.	Verify Gauss Divergence theorem for $\vec{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	CO1	(10)
52.	Find an analytic function $f(z)$ whose real part is $u = e^{-x}(x \sin y - y \cos y)$. Also find its harmonic conjugate 'v'.	CO2	(10)
53.	Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ using contour integration.	CO3	(10)

ALL THE BEST